

# How far is it to a sudden future singularity of pressure?

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We discuss the constraints coming from current observations of type Ia supernovae on cosmological models which allow sudden future singularities of pressure (with the scale factor and the energy density regular). We show that such a sudden singularity may happen in the very near future (e.g. within ten million years) and its prediction at the present moment of cosmic evolution cannot be distinguished, with current observational data, from the prediction given by the standard quintessence scenario of future evolution. Fortunately, sudden future singularities are characterized by a momentary peak of infinite tidal forces only; there is no geodesic incompleteness which means that the evolution of the universe may eventually be continued throughout until another “more serious” singularity such as Big-Crunch or Big-Rip.

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Over the past decade observations of high-redshift Type Ia supernovae (SNIa) have provided strong evidence that the expansion of the universe is accelerating, driven in the standard paradigm by some form of dark energy [1, 2]. Current data [2] continue to leave open the possibility that dark energy exists in the form of phantom energy, which may violate all energy conditions [3]: the null ( $\rho c^2 + p \geq 0$ ), weak ( $\rho c^2 \geq 0$  and  $\rho c^2 + p \geq 0$ ), strong ( $\rho c^2 + p \geq 0$  and  $\rho c^2 + 3p \geq 0$ ), and dominant energy ( $\rho c^2 \geq 0$ ,  $-\rho c^2 \leq p \leq \rho c^2$ ) conditions (where  $c$  is the speed of light,  $\rho$  is the mass density in  $\text{kgm}^{-3}$  and  $p$  is the pressure). Phantom matter may dominate the universe in the future and drive it towards a Big-Rip (BR) singularity in which all matter will be dissociated by gravity [4]. This is dramatically different from the standard picture of future cosmic evolution which suggests an asymptotically empty de-Sitter state driven by the cosmological constant or quintessence [5] and leading to the violation of the strong energy condition only.

Phantom-driven scenarios have encouraged the study of other exotic possibilities for the future evolution of the universe. One of these possibilities appears in those models which do not assume any explicit form for the equation of state  $p = p(\rho)$ , leaving the evolution of the energy density and pressure unconstrained. This freedom may result in a so-called *sudden future singularity* (SFS) of pressure [6] which violates only the dominant energy condition. The nature of a sudden future singularity is different from that of a standard Big-Bang (BB) singularity, and also from a Big-Rip singularity, in that it does not exhibit geodesic incompleteness and the cosmic evolution may eventually be extended beyond it [7, 8]. The only physical characteristic of these singularities is

a momentarily infinite peak of the tidal forces in the universe. In more general models this peak may also appear in the derivatives of the tidal forces. It is interesting to note that these types of singularity are in a way similar to yet another type, which were termed finite density singularities [9]. However, the crucial difference is that finite density singularities occur as singularities in space rather than in time, which means that even at the present moment of cosmic evolution they could exist somewhere in the Universe [10]. We will not discuss in detail finite density singularities in this paper since they basically appear in cosmological models without homogeneity. On the other hand, it is worth mentioning that the sudden future singularities are quite generic since they may arise in both homogeneous [11] and inhomogeneous [12] models of the universe.

In order to obtain a sudden future singularity consider the simple framework of an Einstein-Friedmann cosmology governed by the standard field equations

$$\rho = \frac{3}{8\pi G} \left( \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} \right), \quad (1)$$

$$p = -\frac{c^2}{8\pi G} \left( 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} \right), \quad (2)$$

where the energy-momentum conservation law

$$\dot{\rho} = -3\frac{\dot{a}}{a} \left( \rho + \frac{p}{c^2} \right), \quad (3)$$

is trivially fulfilled due to the Bianchi identity. Here  $a(t)$  is the scale factor,  $G$  is the gravitational constant, and the curvature index  $k = 0, \pm 1$ . What is crucial to obtain a sudden future singularity is that no link between

the energy density and pressure (the equation of state) is specified. This allows us to integrate (3) only by quadratures as

$$\rho a^3 = \exp \left[ - \left( \frac{3p(t')}{c^2 \rho(t')} \ln a(t') \right) \right]_{t_0}^t + \frac{3}{c^2} \int_{t_0}^t \left( \frac{p(t')}{\rho(t')} \right) \ln a(t') dt' \quad (4)$$

Of course (4) reduces to the standard expression for energy conservation,  $\rho a^{3(w+1)} = \text{const.}$ , provided a barotropic equation of state,  $p = w\rho c^2$  for constant  $w$ , is assumed. (The condition for phantom models, for example, is  $w < -1$ ).

From equations (1)-(2) one can easily see that a pressure singularity  $p \rightarrow \mp\infty$  occurs when the acceleration  $\ddot{a} \rightarrow \pm\infty$ , no matter that the value of the energy density  $\rho$  and the scale factor  $a(t)$  are regular. Since in that case  $|p| > \rho$ , it is clear that the dominant energy condition is violated. This condition can be achieved if the scale factor takes the form [6]

$$a(t) = a_s [1 + (1 - \delta) y^m - \delta (1 - y)^n] \quad , \quad y \equiv \frac{t}{t_s} \quad (5)$$

with the appropriate choice of the constants  $\delta, t_s, a_s, m, n$ . Moreover, we can see that the  $r$ -th derivative of the scale factor (5) is given by

$$a^{(r)} = a_s \left[ \frac{m(m-1)\dots(m-r+1)}{t_s^r} (1-\delta) y^{m-r} + (-1)^{r-1} \delta \frac{n(n-1)\dots(n-r+1)}{t_s^r} (1-y)^{n-r} \right] \quad (6)$$

and is related to the appropriate pressure derivative  $p^{(r-2)}$ . Thus, in general, it is possible that one has a pressure derivative  $p^{(r-2)}$  singularity which accompanies the blow-up of the  $r$ -th derivative of the scale factor  $a^{(r)}$ . Observationally this could be manifested in, for example, the blow-up of the characteristics known as *statefinders*, such as jerk, snap etc. [14]. The pressure derivative singularity  $p^{(r-2)}$  appears when

$$r-1 < n < r \quad r = \text{integer} \quad , \quad (7)$$

and for any  $r \geq 3$  it fulfills all energy conditions. These singularities are called generalized sudden future singularities (GSFS) and are possible, for example, in theories with higher-order curvature quantum corrections [13].

Let us now return to the case of  $r = 2$ , for which  $1 < n < 2$  and we obtain sudden future singularity models of pressure (and obviously all of its higher derivatives) which lead to violation of the dominant energy condition. In such models, expressed in terms of the scale factor (5), the evolution begins with the standard BB singularity at  $t = 0$  for  $a = 0$ , and finishes at SFS for  $t = t_s$  where  $a = a_s \equiv a(t_s)$  is a constant. (Note that we have changed the

original parametrization of Ref. [6] for the scale factor (5) using  $A = \delta a_s$ ).

The standard Friedmann limit (i.e. models without an SFS) of (5) is achieved when  $\delta \rightarrow 0$ ; hence  $\delta$  becomes the “non-standardity” parameter of SFS models. Additionally, notwithstanding Ref. [6] and in agreement with the field equations (1)-(2), we assume that  $\delta$  can be both positive and negative leading to a deceleration or an acceleration (cf. (6)) of the universe, respectively.

It is important to our discussion that the asymptotic behaviour of the scale factor (5) close to the BB singularity at  $t = 0$  is given by a simple power-law  $a_{BB} = y^m$ , simulating the behaviour of flat  $k = 0$  barotropic fluid models with  $m = 2/[3(w+1)]$ . This allows us to preserve all the standard observed characteristics of early universe cosmology – such as the cosmic microwave background, density perturbations, nucleosynthesis etc. – provided we choose an appropriate value of  $m$ . On the other hand, close to an SFS the asymptotic behaviour of the scale factor is non-standard,  $a_{SFS} = a_s [1 - \delta (1 - y)^n]$ , showing that  $a_{SFS} = a_s$  for  $t = t_s$  (i.e.  $y = 1$ ) at the SFS. Notice that one does not violate the energy conditions if the parameter  $m$  lies in the range

$$0 < m \leq 1 \quad (w \geq -1/3), \quad (8)$$

This range of values is, in fact, equivalent to a standard (neither quintessence-like nor phantom-like) evolution of the universe. However, with no adverse impact on the field equations (1)-(2), one could also extend the values of  $m$  to lie in the complementary ranges [7]  $m > 1$  (i.e.  $-1 < w < -1/3$ ) for quintessence, and  $m < 0$  (i.e.  $w < -1$ ) for phantom, although these ranges may lead to violation of the strong and weak energy conditions respectively.

We will next calculate the luminosity distance as a function of redshift, and hence the redshift-magnitude relation, for SFS models. This will allow us to establish whether these models are a realistic possibility for the future evolution of the universe, and more specifically whether current cosmological observations of high redshift supernovae are consistent with values of the constant  $n$  in the range  $1 < n < 2$ , as required in order that the scale factor will display an SFS (or, more generally, a GSFS for  $r-1 < n < r$ ). We will then explore the range of values for the other SFS model parameters which are consistent with current observational constraints on standard cosmology, and thus determine limits on how far into the future an SFS might occur. In fact, as we will see below, we need to consider only two further parameters:  $\delta$  and  $y_0 = t_0/t_s$ , where  $t_0$  is the current age of the Universe in the SFS model. Notice that, in view of (8), it is reasonable to take  $m = 2/3$  as for the standard dust-dominated evolution. This implies that, at early times, our SFS model reduces to the Einstein-de-Sitter universe.

We proceed within the framework of Friedmann cosmology, and consider an observer located at  $r = 0$  at coordinate time  $t = t_0$ . The observer receives a light ray emitted at  $r = r_1$  at coordinate time  $t = t_1$ . We then have a standard null geodesic equation

$$\int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_1}^{t_0} \frac{cdt}{a(t)}, \quad (9)$$

with the scale factor  $a(t)$  given by (5). Using (5) again, the redshift is given by

$$1 + z = \frac{a(t_0)}{a(t_1)} = \frac{\delta + (1 - \delta)y_0^m - \delta(1 - y_0)^n}{\delta + (1 - \delta)y_1^m - \delta(1 - y_1)^n}, \quad (10)$$

where  $y_0 = y(t_0)$  and  $y_1 = y(t_1)$ . The luminosity distance is defined as

$$D_L = r_1 a(t_0) (1 + z). \quad (11)$$

Neglecting extinction and  $k$ -corrections, the observed and absolute magnitudes of a source at redshift  $z$  and luminosity distance  $D_L$  are related by

$$m(z) = M - 5 \log_{10} H_0 + 25 + 5 \log_{10} D_L(z), \quad (12)$$

which, with the help of the equation (9), (10) and (11), allows a redshift-magnitude relation for SFS cosmological models to be constructed. It is obvious that equation (9) has to be integrated numerically in order to establish the relation between  $t_0$  and  $t_1$ , which can then be inserted into (10) and (11) to constrain the SFS model parameters. As a first step we determine the dependence on the SFS model parameters of the Hubble law, which replaces equation (12) for  $z \approx 0$ , i.e.  $cz \approx H_0 D_L$ , where

$$\begin{aligned} H_0 (\text{kms}^{-1} \text{Mpc}^{-1}) &= \left( \frac{\dot{a}}{a} \right)_0 \\ &= \frac{3.09 \times 10^{19}}{t_0 (\text{sec}) y_0} \times \left[ \frac{m(1 - \delta)y_0^{m-1} + n\delta(1 - y_0)^{n-1}}{\delta + (1 - \delta)y_0^m - \delta(1 - y_0)^n} \right] \end{aligned} \quad (13)$$

is the present value of the Hubble parameter, which we can take as  $72 \text{kms}^{-1} \text{Mpc}^{-1}$  [1].

Similarly we could derive an expression, in terms of the SFS model parameters, for the deceleration parameter  $q_0 = -(\ddot{a}/\dot{a}^2)_0$ . However, in order to search the parameter space for models which are admissible by current observations, we write the product of  $H_0$  and  $q_0$  as

$$\begin{aligned} q_0 H_0 &= - \left( \frac{\ddot{a}}{\dot{a}} \right)_0 \\ &= - \frac{t_0}{y_0} \frac{m(m-1)(1-\delta)y_0^{m-2} - \delta n(n-1)(1-y_0)^{n-2}}{m(1-\delta)y_0^{m-1} + n\delta(1-y_0)^{n-1}}. \end{aligned} \quad (14)$$

In order to obtain an accelerated universe at the present moment of the evolution, this product should be negative. Fig. 1 shows an example plot of the product  $H_0 q_0$

as a function of  $\delta$  and  $y_0$ , with the other parameters fixed at  $m = 2/3$ ,  $n = 1.9993$ ,  $t_0 = 13.2457$  Gyr. From the plot we see that there are large regions of the parameter space which admit cosmic acceleration. We have explored the parameter space further with various configurations of  $m, n, \delta, y_0, t_0, q_0$ , and  $H_0$ , and obtained the general conclusion that there is a large class of SFS models which are compatible with current acceleration.

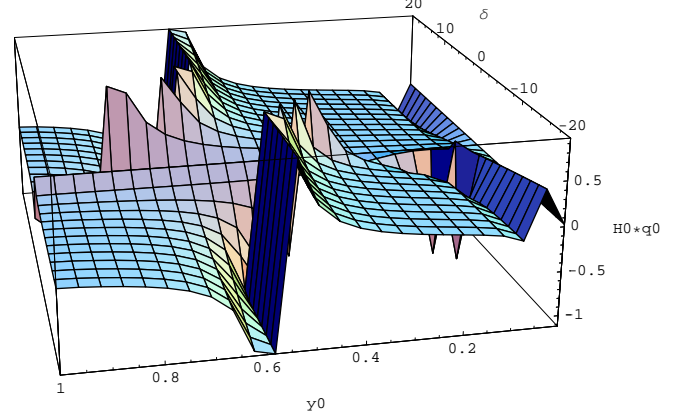


FIG. 1: Parameter space  $(H_0 q_0, \delta, y_0)$  for fixed values of  $m = 2/3$ ,  $n = 1.9993$ ,  $t_0 = 13.3547$  Gyr of the sudden future singularity models. There are large regions of the parameter space which admit cosmic acceleration.

Out of these admissible models we then searched for those which are compatible with the redshift-magnitude relation (12) observed for recent SNIa data [2], and hence with the derived parameters of the standard ‘Concordance cosmology’ (CC). We were able to identify SFS models that are in remarkably tight agreement with current SNIa data. As an illustrative example Fig. 2 shows luminosity distance as a function of redshift for the CC model with  $H_0 = 72 \text{kms}^{-1} \text{Mpc}^{-1}$ ,  $\Omega_{m0} = 0.26$  and  $\Omega_{\Lambda 0} = 0.74$ , and an SFS model with parameters  $m = 2/3$ ,  $y_0 = 0.99936$ ,  $\delta = -0.471$ ,  $n = 1.9999$ . We see that the SFS model mimics the CC model very closely over a wide range of redshifts. In particular, it is clear that recent SNIa data from the Tonry *et al.* ‘Gold’ sample [1] and SNLS sample [2] cannot yet discriminate between the CC and SFS models.

Taking the current age of the universe in the SFS model to be equal to the age of the CC model, i.e.  $t_0 = 13.6$  Gyr, we find that the time to the sudden singularity is  $t_s - t_0 \approx 8.7$  Myr, which is *amazingly* close to the present epoch. In that context there is no wonder that these singularities are called “sudden”. We have also checked that the larger the value of  $r$  in (7) the later in future a GSFS appears. It means that the strongest of these singularities which violates the dominant energy condition (i.e. an SFS) is more likely to become reality.

Our remark about the effect of the sudden pressure singularity seems in agreement with the result of Ref.

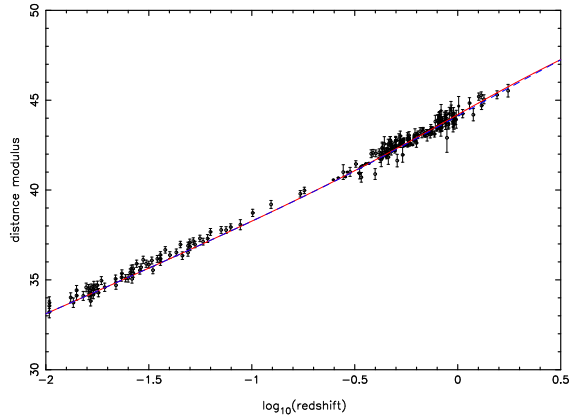


FIG. 2: The distance modulus  $\mu_L = m - M$  for the concordance cosmology (CC) model with  $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_{m0} = 0.26$ ,  $\Omega_{\Lambda 0} = 0.74$  (dashed curve) and sudden future singularity (SFS) model for  $m = 2/3$ ,  $n = 1.9999$ ,  $\delta = -0.471$ ,  $y_0 = 0.99936$  (solid curve). Also shown are the ‘Gold’ (open circles) and SNLS (filled circles) SNIa data. Taking the age of the SFS model to be equal to that of the CC model, i.e.  $t_0 = 13.6$  Gyr, one finds that an SFS is possible in only 8.7 million years.

[16] which showed that the dominant energy condition is now violated and that it became violated quite recently (at redshift  $z \sim 0.2$ ). Of course this violation may also be due to phantom energy [3].

In conclusion, we have shown that a sudden future singularity may happen in the comparatively near future (e.g. within ten million years) and its prediction at the present moment of cosmic evolution cannot be distinguished, with current observational data, from the prediction given by the standard quintessence scenario of future evolution in the Concordance Model. Fortunately, sudden future singularities are characterized by a momentary peak of infinite tidal forces only; there is no geodesic incompleteness which means that the evolution of the universe may eventually be continued beyond the SFS until another “more serious” singularity such as a Big-Crunch or a Big-Rip. One could then consider, more generally, a scale factor of the form [7, 15]

$$a(t) = A + [(a_s - A) - D(t_r - t_s)^p - Et_s^o] y^m \quad (15) \\ - (A + Dt_r^p)(1 - y)^n + D(t_r - t_s y)^p + Et_s^o y^o,$$

where the constants  $m, o, p, A, D, E$  are chosen so that the universe begins with a Big-Bang at  $t = 0$  where  $a = 0$ , next faces a sudden future singularity at  $t = t_s$  where  $a(t_s) = a_s$ , and then eventually continues to a Big-Rip at  $t = t_r$  where  $a(t_r) \rightarrow \infty$ . All of the matter sources may be involved since the constants in (15) can be taken as:  $0 < m \leq 1$  (quintessence),  $p < 0$  (phantom), and  $o > 1$  (standard positive matter pressure).

Whether the universe will end in a Big-Rip or a Big-Crunch is an open question. Moreover, unlike a sudden

future singularity, both a Big-Rip and Big-Crunch singularity would represent the real end of the universe. Fortunately, as was shown in Refs. [4, 17], a Big-Rip singularity is not possible in the very near future: in order to reach it one must wait about the same time as the current age of the universe. Apart from that, it is still possible to avoid it due to a negative tension brane contribution in a turnaround cyclic cosmology [18].

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